

Semester 1 (Unit 3) Examination, 2017

Question/Answer Booklet

MATHEMATICS SPECIALIST

Section Two: Calculator-assumed

Student Name/Number: _____

Teacher Name: _____

Time allowed for this section

Reading time before commencing work: five minutes

Working time for this section: one-hundred minutes

Materials required/recommended for this section

To be provided by the supervisor: This Question/Answer Booklet
Formula Sheet

To be provided by the candidate:

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on **two** unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	10	10	100	92	65
					100

Instructions to candidates

- The rules for the conduct of School exams are detailed in the _____ *School/College assessment policy*.
Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer Booklet.
- You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
- Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
- Show all working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil**, except in diagrams.
- The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed

(92 Marks) Weighting 65%

This section has **10** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.

- Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
- Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Suggested working time: **100 minutes**.

Question 8**(8 marks)**

The line ℓ passes through the points $P(2, -3, 5)$ and $Q(5, 1, -3)$

(a) Find a vector equation of the line ℓ . (3 marks)

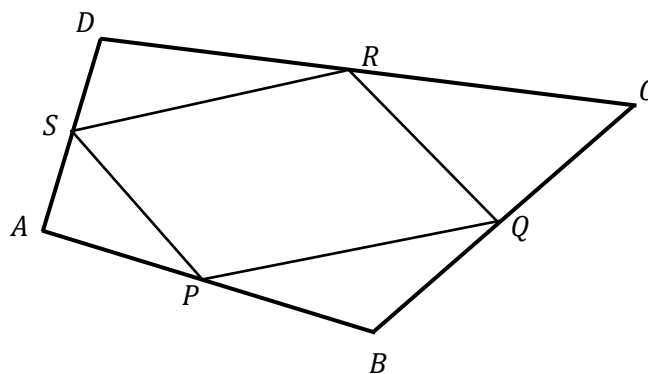
(b) Write down a set of parametric equations for ℓ . (2 marks)

(c) Does the point $R(8, 5, 0)$ lie on ℓ ? Justify your answer. (3 marks)

Question 9

(10 marks)

- (a) The mid-points of the sides of the quadrilateral $ABCD$ are the points P, Q, R and S , as shown below.



Use a vector proof to show that the mid-points of any quadrilateral are the vertices of a parallelogram.

Question 9 (continued)

(b) Given $P(x, y, z)$ is a point not on line L . Let line L pass through points $A(0,1,1)$ and

$B(2,0,1)$. If $\overrightarrow{AB} = \mathbf{n}$ and $\overrightarrow{AP} = \mathbf{m}$, show that $\mathbf{n} \times \mathbf{m} = \begin{pmatrix} 1-z \\ 2-2z \\ x+2y-2 \end{pmatrix}$ (5 marks)

Question 10**(10 marks)**

The polynomial $P(x) = (x^2 + 2x + b)(x - a)$, $a \geq b \geq 0$, and a, b are integers.

(a) If $a = 6$, state all possible real roots of $P(x) = 0$. (5 marks)

(b) Determine $P(x)$ in terms of its linear factors, in the form $x - (\alpha + \beta i)$ for all values of $2 \leq b \leq 5$ and α, β rational. (5 marks)

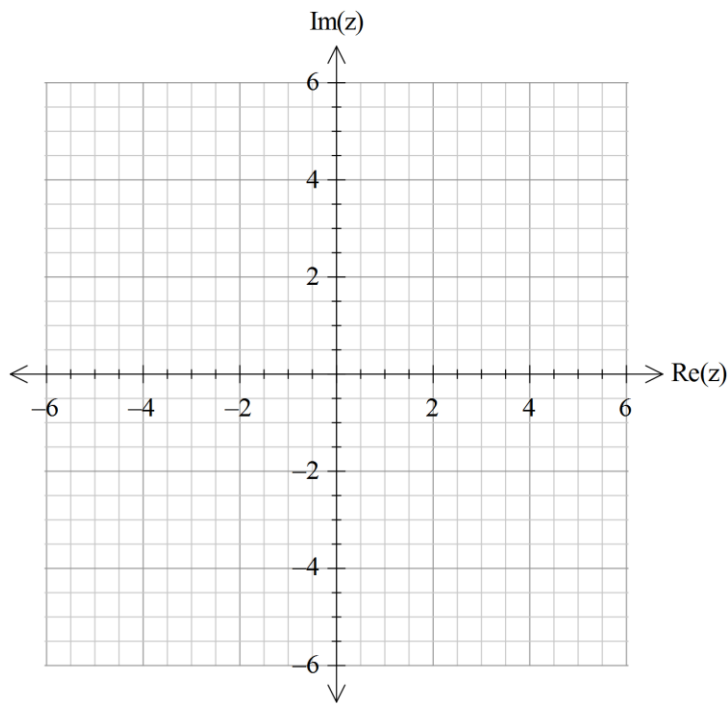
Question 11

(11 marks)

Sketch each of the following solution sets of complex numbers on the argand diagrams provided.

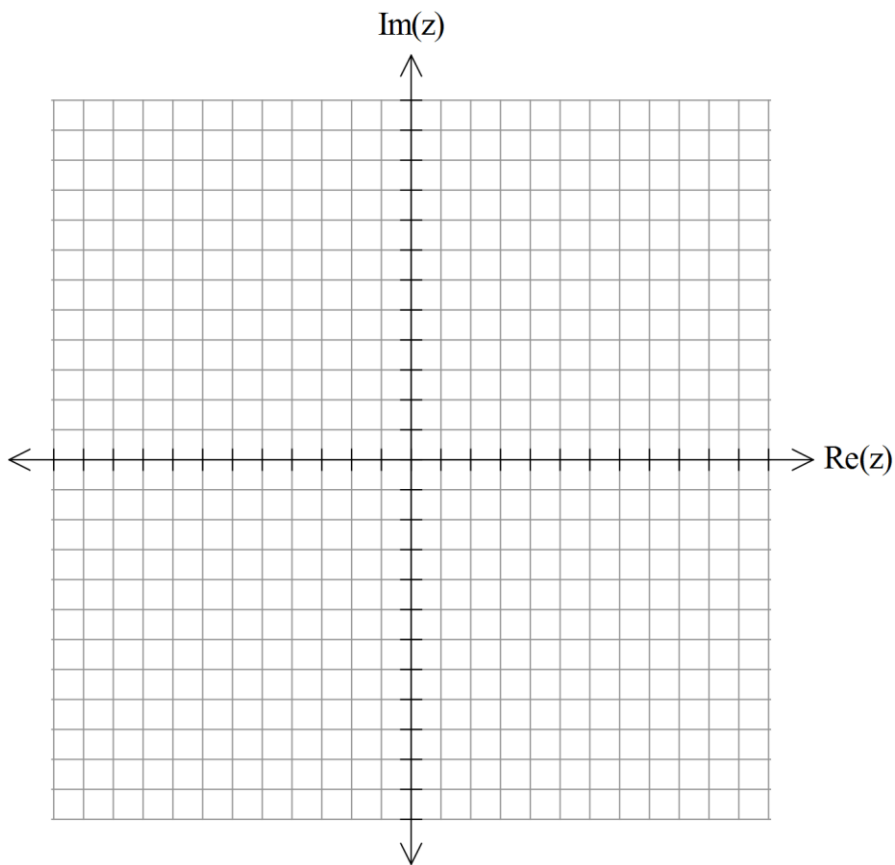
(a) $|z - 2i| = |z - 2|$

(3 marks)



(b) $\text{Re}(z) < 2, \text{Im}(z) > -2, 2 < |z| < 5$ and $-\frac{\pi}{4} \leq \text{Arg}(z) \leq \frac{2\pi}{3}$

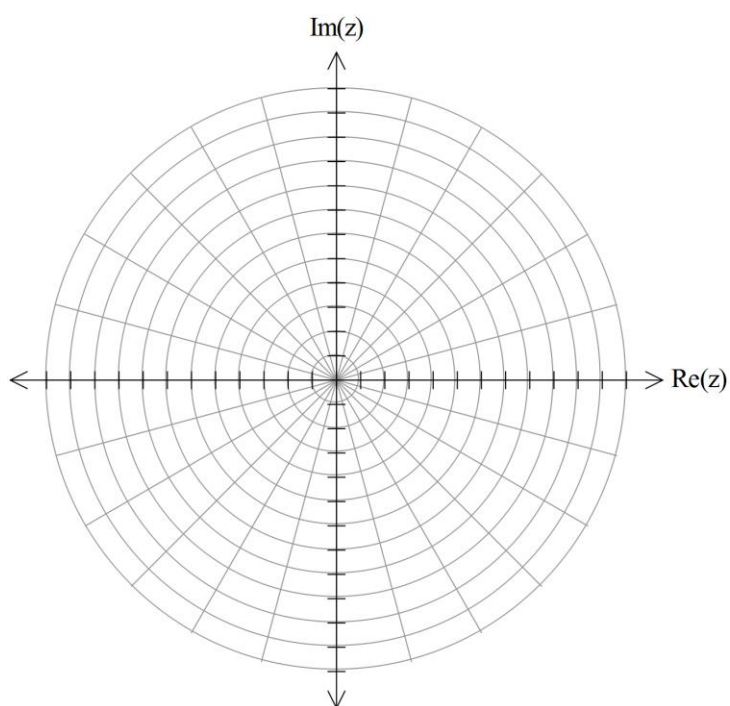
(8 marks)



Question 12

(5 marks)

Determine the 5th roots of $-32i$ and plot them on the Argand plane provided below.



Question 13**(6 marks)**

Given $2\bar{z} + z\bar{z} = 24 + 8i$, determine z .

Question 14**(7 marks)**

The plane \mathcal{P} intersects the coordinate axes at the points $A(-3,0,0)$, $B(0,4,0)$ and $C(0,0,1)$.

(a) Use a cross product to find a vector \mathbf{n} that is normal to the plane \mathcal{P} (2 marks)

(b) Find a Cartesian equation for \mathcal{P} . (2 marks)

A vector equation for the line \mathcal{L} is $\mathbf{r} = (3 - 3t)\mathbf{i} + (5 + 2t)\mathbf{j} + t\mathbf{k}$.

(c) Find the coordinates of D , the point of intersection of the line \mathcal{L} and plane \mathcal{P} . (3 marks)

Question 15**(16 marks)**

The position vector $\mathbf{r}(t)$ of a model railway train at time t , in an appropriately chosen coordinate system, is given by

$$\mathbf{r}(t) = 3 \cos kt \mathbf{i} - 2 \sin kt \mathbf{j}$$

where distances are measured in metres and time is measured in seconds after an appropriately chosen starting time. The number k is positive.

- (a) Obtain a Cartesian equation for the path traversed by the train. (3 marks)
- (b) Describe the geometric shape of the path of the train. (1 mark)
- (c) Obtain expressions for $\mathbf{v}(t)$ and $\mathbf{a}(t)$, the velocity and acceleration at time t . (2 marks)
- (d) Does the train travel in a clockwise or anticlockwise direction around its closed path? Justify your answer. (2 marks)

- (e) At which points on the path is the velocity perpendicular to the acceleration? (3 marks)
- (f) Evaluate k , correct to 3 significant figures, given that the train completes a circuit every 92 seconds. (2 marks)
- (g) What is the maximum speed of the train? Justify your answer. (3 marks)

Question 16**(10 marks)**

The Cartesian equation of the sphere \mathcal{S} is

$$x^2 - 6x + y^2 + z^2 + 10z = 2.$$

- (a) Determine the radius and the coordinates of the centre of the sphere \mathcal{S} . (3 marks)

The vector equation of the line ℓ is

$$\mathbf{r}(t) = (9 + 2t)\mathbf{i} - 2t\mathbf{j} + (1 + t)\mathbf{k}$$

- (b) Does the line ℓ intersect the sphere \mathcal{S} , and if so, where? (5 marks)

(c) Explain why ℓ is tangential to \mathcal{S} .

(2 marks)

Question 17

(9 marks)

(a) Solve for $|x-2| \geq |x+8|$

(2 marks)

(b) Determine the value of the constant a if $|2x-a| \leq |x-7|$ is only true for $-5 \leq x \leq 3$.

(3 marks)

(c) Determine the values of the constants p, q and s such that $|3x-2| = p|x+q| + s$ is only true for $\frac{2}{3} \leq x \leq 2$.

(4 marks)

End of Questions

Additional working space

Question number: _____

Acknowledgements

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*Published by The Mathematical Association of WA
12 Cobbler Place, MIRRABOOKA 6061.*